2 POLYNOMIALS

EXERCISE 2.1

Q.1. *Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.*

(i) $4x^2 - 3x + 7$ *(ii)* $y^2 + \sqrt{2}$ *(iii)* $3\sqrt{t} + t\sqrt{2}$ *(iv)* $y + \frac{2}{y}$ *(v)* $x^{10} + y^3 + t^{50}$

Sol. (i) Polynomial in one variable, x **Ans.**

(ii) Polynomial in one variable, y. **Ans.**

(iii) $3\sqrt{t} + t\sqrt{2}$ is not a polynomial as power of t in $\sqrt{t}$ is not a whole number. **Ans.**

(iv) $y + \frac{2}{y}$ is not a polynomial as power of y in second term, i.e., $\frac{1}{y} = y^{-1}$ is not a whole number. **Ans.**

(v) $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable but a polynomial in three variables x, y and t. **Ans.**

Q.2. *Write the coefficients of x^2 in each of the following :*

(i) $2 + x^2 + x$ *(ii)* $2 - x^2 + x^3$ *(iii)* $\frac{\pi}{2}x^2 + x$ *(iv)* $\sqrt{2}x - 1$

Sol. (i) In $2 + x^2 + x$, coefficient of x^2 is 1. **Ans.**

(ii) In $2 - x^2 + x^3$, coefficient of x^2 is -1. **Ans.**

(iii) $\frac{\pi}{2}x^2 + x$, coefficient of x^2 is $\frac{\pi}{2}$. **Ans.**

(iv) $\sqrt{2}x - 1$, x^2 is not present hence no coefficient. **Ans.**

Q.3. *Give one example each of a binomial of degree 35, and of a monomial of degree* 100.

Sol. $x^{35} + 5$ is a binomial of degree 35.

$2y^{100}$ is a monomial of degree 100. **Ans.**

Q.4. *Write the degree of each of the following polynomials :*

(i) $5x^3 + 4x^2 + 7x$ *(ii)* $4 - y^2$ *(iii)* $5t - \sqrt{7}$ *(iv)* 3

Sol. (i) Degree is 3 as x^3 is the highest power. **Ans.**

(ii) Degree is 2 as y^2 is the highest power. **Ans.**

(iii) Degree is 1 as t is the highest power. **Ans.**

(iv) Degree is 0 as x^0 is the highest power. **Ans.**

Q.5. *Classify the following as linear, quadratic and cubic polynomials :*

(i) $x^2 + x$ *(ii)* $x - x^3$ (iii) $y + y^2 + 4$

(iv) $1 + x$ *(v)* $3t$ (vi) r^2 (vii) $7x^3$

Sol. (i) $x^2 + x$ is quadratic. (ii) $x - x^3$ is cubic.

(iii) $y + y^2 + 4$ is quadratic. (iv) $1 + x$ is linear.

(v) $3t$ is linear. (vi) r^2 is quadratic.

(vii) $7x^3$ is cubic.

2 POLYNOMIALS

EXERCISE 2.2

Q.1. *Find the value of the polynomial* $5x - 4x^2 + 3$ *at*

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Sol. $p(x) = 5x - 4x^2 + 3$

(i) At $x = 0$, $p(0) = 5 \times 0 - 4 \times 0^2 + 3 = 3$ **Ans.**

(ii) At $x = -1$, $p(-1) = 5 \times (-1) - 4 \times (-1)^2 + 3 = -5 - 4 + 3 = -6$ **Ans.**

(iii) At $x = 2$, $p(2) = 5 \times 2 - 4 \times (2)^2 + 3 = 10 - 16 + 3 = -3$ **Ans.**

Q.2. *Find* $p(0)$, $p(1)$ *and* $p(2)$ *for each of the following polynomials :*

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Sol. (i) $p(y) = y^2 - y + 1$

$p(0) = 0^2 - 0 + 1 = 1$

$p(1) = 1^2 - 1 + 1 = 1$

$p(2) = 2^2 - 2 + 1 = 3$. **Ans.**

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2 \times 0^2 - 0^3 = 2$

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3 = 4$

$p(2) = 2 + 2 + 2 \times 2^2 - 2^3 = 4 + 8 - 8 = 4.$

(iii) $p(x) = x^3$

$p(0) = 0$

$p(1) = 1$

$p(2) = 8$. **Ans.**

(iv) $p(x) = (x - 1)(x + 1)$

$p(0) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = 0$

$p(2) = (2 - 1)(2 + 1) = 3$ **Ans.**

Q.3. *Verify whether the following are zeroes of the polynomial, indicated against them.*

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, x = 1, -1$

(iv) $p(x) = (x + 1)(x - 2), x = -1, 2$

(v) $p(x) = x^2, x = 0$ (vi) $p(x) = lx + m, x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Sol. (i) Yes. $3x + 1 = 0$, for $x = -\frac{1}{3}$. **Ans.**

(ii) No. $5x - \pi = 5 \times \frac{4}{5} - \pi = 4 - \pi \neq 0$ **Ans.**

(iii) Yes. $x^2 - 1 = 1^2 - 1 = 0$ for $x = 1$

and $x^2 - 1 = (-1)^2 - 1 = 0$ for $x = 1$ **Ans.**

(iv) Yes. $(x + 1)(x - 2) = 0$ for $x = -1$, or, $x = 2$.

(v) Yes. $x^2 = 0$ for $x = 0$

(vi) Yes. $lx + m = 0$ for $x = -\frac{m}{l}$

(vii) $3x^2 - 1 = 3\frac{1}{3} - 1 = 0$ for $x = \frac{-1}{\sqrt{3}}$

and $3x^2 - 1 = 3\cdot\frac{4}{3} - 1 = 3 \neq 0$

Thus, for $\frac{-1}{\sqrt{3}}$ is a zero but $-\frac{2}{\sqrt{3}}$ is not a zero of the polynomial **Ans.**

(viii) No. $2x + 1 \neq 0$ for $x = \frac{1}{2}$.

Q.4. *Find the zero of the polynomial in each of the following cases :*

(i) $p(x) = x + 5$ *(ii)* $p(x) = x - 5$

(iii) $p(x) = 2x + 5$ *(iv)* $p(x) = 3x - 2$

(v) $p(x) = 3x$ *(vi)* $p(x) = ax$, $a \neq 0$

(vii) $p(x) = cx + d$, $c \neq 0$, c, d are real numbers.

Sol. (i) $x + 5 = 0$, $x = -5$, so, -5 is the zero of $x + 5$ **Ans.**

(ii) $x - 5 = 0$, $x = 5$ so, 5 is the zero of $x - 5$ **Ans.**

(iii) $2x + 5 = 0$, $\Rightarrow 2x = -5$,

$\Rightarrow x = \frac{-5}{2}$, so $-\frac{5}{2}$ is the zero of $2x + 5$ **Ans.**

(iv) $3x - 2 = 0 \Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$, so $\frac{2}{3}$ is the zero of $3x - 2$ **Ans.**

(v) $3x = 0$, $\Rightarrow x = 0$, so 0 is the zero of $3x$ **Ans.**

(vi) $ax = 0$ $(a \neq 0) \Rightarrow x = \frac{0}{a} = 0$, so, is the zero of ax **Ans.**

(vii) $cx + d = 0$ $(c \neq 0)$

$\Rightarrow cx = -d$

$\Rightarrow x = \frac{-d}{c}$, so, $\frac{-d}{c}$ is the 0 of $cx + d$ **Ans.**

2 POLYNOMIALS

EXERCISE 2.3

Q.1. *Find the remainder when* $x^3 + 3x^2 + 3x + 1$ *is divided by*

(i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) x

(iv) $x + \pi$ (v) $5 + 2x$

Sol. $p(x) = x^3 + 3x^2 + 3x + 1$

(i) When $p(x)$ is divided by $x + 1$,

i.e., $x + 1 = 0$, $x = -1$ is to be substituted in $p(x)$.

$$p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$$

$$= -1 + 3 - 3 + 1 = 0$$

Remainder = 0. **Ans.**

(ii) When $p(x)$ is divided by $x - \frac{1}{2}$ remainder is $p\left(\frac{1}{2}\right)$.

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1 + 6 + 12 + 8}{8}$$

$\therefore$ Remainder $= \frac{27}{8} = 3\frac{3}{8}$ **Ans.**

(iii) When $p(x)$ is divided by x, then remainder is $p(0)$.

$x = 0$, substitute in $p(x)$

$$p(0) = 0^3 + 3 \times 0^2 + 3 \times 0 + 1 = 1.$$

$\therefore$ Remainder = 1 **Ans.**

(iv) Wgeb $p(x)$ is divided by $x + \pi$, then, remainder is $p(-\pi)$.

$x = -\pi$ to be substituted in $p(x)$

$$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1.$$

$\therefore$ Remainder $= -\pi^3 + 3\pi^2 - 3\pi + 1$ **Ans.**

(v) When $p(x)$ is divided by $(5 + 2x)$, then remainder is $p\left(\frac{-5}{2}\right)$.

$$p\left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$$

$$= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = \frac{-125 + 150 - 60 + 8}{8}$$

Remainder $= \frac{-35 + 8}{8} = \frac{-27}{8}$ **Ans.**

Q.2. *Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.*

Sol. $p(x) = x^3 - ax^2 + 6x - a$

When $p(x)$ is divided by $x - a$, the remainder is $p(a)$.

Substitute $x = a$ in $p(x)$

$p(a) = a^3 - a^3 + 6a - a = 5a$ **Ans.**

Q.3. *Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.*

Sol. $7 + 3x = 0$

$\Rightarrow \quad 3x = -7$

$\Rightarrow \quad x = \dfrac{-7}{3}$

Substitute $x = \dfrac{-7}{3}$ in $p(x) = 3x^3 + 7x$

$$p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = \frac{-343}{9} - \frac{49}{3} = \frac{-343-147}{9} = \frac{-490}{9}.$$

So, remainder $= \dfrac{-490}{9}$ which is different from 0.

Therefore, $(3x + 7)$ is not a factor of the polynomial $3x^3 + 7x$. **Ans.**

2 POLYNOMIALS

EXERCISE 2.4

Q.1. *Determine which of the following polynomials has $(x + 1)$ a factor :*

(i) $x^3 + x^2 + x + 1$ *(ii)* $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ *(iv)* $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Sol. To have $(x + 1)$ as a factor, substituting $x = -1$ must give $p(-1) = 0$.

(i) $x^3 + x^2 + x + 1$

$= (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

Therefore, $x + 1$ is a factor of $x^3 + x^2 + x + 1$ **Ans.**

(ii) $x^4 + x^3 + x^2 + x + 1$

$= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 - 1 + 1 - 1 + 1 = 1$

Remainder is not 0. Therefore $(x + 1)$ is not its factor. **Ans.**

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

$= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$. Remaidner is not 0

Therefore, $(x + 1)$ is not its factor. **Ans.**

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$= (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2+\sqrt{2}+\sqrt{2} = 2\sqrt{2}$

Remainder not 0, therefore $(x + 1)$ is not a factor. **Ans.**

Q.2. *Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases :*

(i) $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6,\ g(x) = x - 3$

Sol. (i) $g(x) = x + 1$. $x = -1$ to be substituted in

$p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0.$

So, $g(x)$ is a factor of $p(x)$. **Ans.**

(ii) $g(x) = x + 2$, substitute $x = -2$ in $p(x)$

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = -1.$

So, $g(x)$ is not a factor of $p(x)$ **Ans.**

(iii) $g(x) = x - 3$ substitute $x = 3$ in (x).

$p(x) = x^3 - 4x^2 + x + 6$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6 = 27 - 36 + 3 + 6 = 0.$

Therefore, $g(x)$ is a factor of $x^3 - 4x^2 + x + 6$. **Ans.**

Q.3. *Find the value of k, if x – 1 is a factor of p(x) in each of the following cases :*

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ *(iv)* $p(x) = kx^2 - 3x + k$

Sol. $(x - 1)$ is a factor, so we substitute $x = 1$ in each case and solve for k by making $p(1)$ equal to 0.

(i) $p(x) = x^2 + x + k$

$p(1) = 1 + 1 + k = 0 \Rightarrow k = -2$ **Ans.**

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 2 \times 1^2 + k \times 1 + \sqrt{2} = 0$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$\Rightarrow k = -2 - \sqrt{2} = -(2 + \sqrt{2})$ **Ans.**

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

$p(1) = k - \sqrt{2} + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$ **Ans.**

(iv) $p(x) = kx^2 - 3x + k$

$p(1) = k - 3 + k = 0 \Rightarrow 2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$ **Ans.**

Q.4. *Factorise :*

(i) $12x^2 - 7x + 1$ *(ii)* $2x^2 + 7x + 3$ *(iii)* $6x^2 + 5x - 6$ *(iv)* $3x^2 - x - 4$

Sol. (i) $12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1) = (4x - 1)(3x - 1)$ **Ans.**

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$ **Ans.**

(iii) $6x^2 + 5x - 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3) = (3x - 2)(2x + 3)$ **Ans.**

(iv) $3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (x + 1)(3x - 4)$ **Ans.**

Q.5. *Factorise :*

(i) $x^3 - 2x^2 - x + 2$ *(ii)* $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ *(iv)* $2y^3 + y^2 - 2y - 1$

Sol. (i) $p(x)\ x^3 - 2x^2 - x + 2$

Let us guess a factor $(x - a)$ and choose value of a arbitrarily as 1.

Now, putting this value in $p(x)$.

$1 - 2 - 1 + 2 = 0$

So $(x - 1)$ is a factor of $p(x)$

Now, $x^3 - 2x^2 - x + 2 = x^3 - x^2 - x^2 + x - 2x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)\{x(x - 2) + 1(x - 2)\}$

$= (x - 1)(x + 1)(x - 2)$ **Ans.**

To factorise it

$x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x + 1)(x - 2)$.

After factorisation $(x - 1)(x + 1)(x - 2)$.

(ii) $p(x) = x^3 - 3x^2 - 9x - 5$

Take a factor $(x - a)$. a should be a factor of 5, i.e., $\pm$ 1 or $\pm$ 5.

For $(x - 1)$, $a = 1$

$$p(1) = (1)^3 - (-3)\ 1^2 - 9 \times 1 - 5$$
$$= 1 - 3 - 9 - 5 = -16.$$

So, $(x - 1)$ is not a factor of $p(x)$.

For $a = 5$

$$p(5) = (5)^3 - 3(5)^2 - 9\ (5) - 5$$
$$= 125 - 75 - 45 - 5 = 0.$$

Therefore, $(x - 5)$ is a factor of $x^3 - 3x^2 - 9x - 5$.

Now, $x^3 - 3x^2 - 9x - 5 = x^3 - 5x^2 + 2x^2 - 10x + x - 5$

$$= x^2\ (x - 5) + 2x\ (x - 5) + 1\ (x - 5)$$
$$= (x - 5)\ (x^2 + 2x + 1)$$
$$= (x - 5)\ (x + 1)^2$$
$$= (x - 5)\ (x + 1)\ (x + 1)$$

So, $x^3 - 3x^2 - 9x - 5 = (x - 5)\ (x + 1)\ (x + 1)$. **Ans.**

(iii) $p(x) = x^3 + 13x^2 + 32x + 20$

Let a factor be $(x - a)$. a should be a factor of 20 which are $\pm$ 1, $\pm$ 2, $\pm$ 4, $\pm$ 5, $\pm$ 10.

For $x - 1 = 0 \Rightarrow x = 1$

Now, $p(1) = 1 + 13 + 32 + 20$

$$= 66 \neq 0$$

Hence, $(x - 1)$ is not a factor of $p(x)$.

Again, for $x + 1 = 0 \quad \Rightarrow x = -1$

Now, $p\ (-1) = -1 + 13 - 32 + 20$

$$= -33 + 33 = 0$$

Hence, $(x + 1)$ is a factors of $p(x)$.

Now, $x^3 + 13x^2 + 32x + 20 = x^3 + x^2 + 12x^2 + 20x + 20$

$$= x^2(x + 1) + 12\ x\ (x + 1) + 20\ (x + 1)$$
$$= (x + 1)\ (x^2 + 12x + 20)$$
$$= (x + 1)\ (x^2 + 10x + 2x + 20)$$
$$= (x + 1)\ \{x\ (x + 10) + 2\ (x + 10)\}$$
$$= (x + 2)\ (x + 1)\ (x + 10)$$ **Ans.**

(iv) $p(y) = 2y^3 + y^2 - 2y - 1$

factors of -2 are $\pm$ 1, $\pm$ 2.

$p(1)\ 2 \times 1^3 + 1^2 - 2 \times 1 - 1$

$= 2 + 1 - 2 - 1 = 0$.

Therefore, $(y - 1)$ is a factor of $p(y)$.

Now, $2y^3 + y^2 - 2y - 1 = 2y^3 - 2y^2 + 3y^2 - 3y + y - 1$

$$= 2y^2\ (y - 1) + 3y\ (y - 1) + 1\ (y - 1)$$
$$= (y - 1)\ (2y^2 + 3y) + 1)$$
$$= (y - 1)\ (2y^2 + 2y + y + 1)$$
$$= (y - 1)\ \{2y\ (y + 1) + 1\ (y + 1)$$
$$= (y - 1)\ (y + 1)\ (2y + 1)$$

Therefore, $2y^3 + y^2 - 2y - 1 = (y - 1)\ (2y + 1)\ (y + 1)$. **Ans.**

2 POLYNOMIALS

EXERCISE 2.5

Q.1. *Use suitable identities to find the following products :*

(i) $(x + 4)(x + 10)$ *(ii)* $(x + 8)(x - 10)$

(iii) $(3x + 4)(3x - 5)$ *(iv)* $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ *(v)* $(3 - 2x)(3 + 2x)$

Sol. (i) Using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x + 4)(x + 10) = x^2 + (4 + 10)x + 4 \times 10 = x^2 + 14x + 40$ **Ans.**

(ii) Using the same identity as in (i) above

$(x + 8)(x - 10) = x^2 + (8 - 10)x + 8 \times (-10) = x^2 - 2x - 80$ **Ans.**

(iii) Using the same identity

$(3x + 4)(3x - 5) = 3x \times 3x + (-1)(3x) - 20 = 9x^2 - 3x - 20.$ **Ans.**

(iv) Using $(x + y)(x - y) = x^2 - y^2$

$\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}$ **Ans.**

(v) Using the same identity as in (iv)

$(3 - 2x)(3 + 2x) = 3^2 - (2x)^2$

$= 9 - 4x^2$ **Ans.**

Q.2. *Evaluate the following products without multiplying directly :*

(i) 103×107 *(ii)* 95×96 *(iii)* 104×96

Sol. (i) $103 \times 107 = (100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7) \times 100 + 3 \times 7$

$= 10000 + 1000 + 21 = 11021$ **Ans.**

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 - (5 + 4) \times 100 + 5 \times 4$

$= 10000 - 900 + 20 = 9120$ **Ans.**

(iii) $104 \times 96 = (100 + 4)(100 - 4) = 100^2 - 4^2$

$= 10000 - 16 = 9984$ **Ans.**