

EXERCISE 8.1

Q.1. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Suppose the measures of four angles are $3x$, $5x$, $9x$ and $13x$.

$$\therefore 3x + 5x + 9x + 13x = 360^\circ \quad [\text{Angle sum property of a quadrilateral}]$$

$$\Rightarrow 30x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{30} = 12^\circ$$

$$\Rightarrow 3x = 3 \times 12^\circ = 36^\circ$$

$$5x = 5 \times 12^\circ = 60^\circ$$

$$9x = 9 \times 12^\circ = 108^\circ$$

$$13x = 13 \times 12^\circ = 156^\circ$$

\therefore the angles of the quadrilateral are **36° , 60° , 108° and 156°** Ans.

Q.2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol. **Given :** ABCD is a parallelogram in which $AC = BD$.

To Prove : ABCD is a rectangle.

Proof : In $\triangle ABC$ and $\triangle BAD$

$$AB = AB \quad [\text{Common}]$$

$$BC = AD \quad [\text{Opposite sides of a parallelogram}]$$

$$AC = BD \quad [\text{Given}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SSS congruence}]$$

$$\angle ABC = \angle BAD \quad \dots(i) \quad [\text{CPCT}]$$

Since, ABCD is a parallelogram, thus,

$$\angle ABC + \angle BAD = 180^\circ \quad \dots(ii)$$

[Consecutive interior angles]

$$\angle ABC + \angle ABC = 180^\circ$$

$$\therefore 2\angle ABC = 180^\circ \quad [\text{From (i) and (ii)}]$$

$$\Rightarrow \angle ABC = \angle BAD = 90^\circ$$

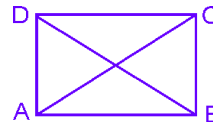
This shows that ABCD is a parallelogram one of whose angle is 90° .

Hence, ABCD is a rectangle. **Proved.**

Q.3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Sol. **Given :** A quadrilateral ABCD, in which diagonals AC and BD bisect each other at right angles.

To Prove : ABCD is a rhombus.



Proof : In $\triangle AOB$ and $\triangle BOC$

$$AO = OC$$

[Diagonals AC and BD bisect each other]

$$\angle AOB = \angle COB \quad [\text{Each} = 90^\circ]$$

$$BO = BO \quad [\text{Common}]$$

$$\therefore \triangle AOB \cong \triangle BOC \quad [\text{SAS congruence}]$$

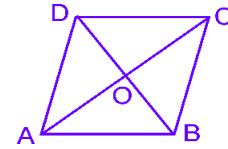
$$AB = BC \quad \dots(i) \quad [\text{CPCT}]$$

Since, ABCD is a quadrilateral in which

$$AB = BC \quad [\text{From (i)}]$$

Hence, ABCD is a rhombus.

[\therefore if the diagonals of a quadrilateral bisect each other, then it is a parallelogram and opposite sides of a parallelogram are equal] **Proved.**



Q.4. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol. Given : ABCD is a square in which AC and BD are diagonals.

To Prove : AC = BD and AC bisects BD at right angles, i.e. $AC \perp BD$.

$$AO = OC, \quad OB = OD$$

Proof : In $\triangle ABC$ and $\triangle BAD$,

$$AB = AB \quad [\text{Common}]$$

$$BC = AD \quad [\text{Sides of a square}]$$

$$\angle ABC = \angle BAD = 90^\circ \quad [\text{Angles of a square}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SAS congruence}]$$

$$\Rightarrow AC = BD \quad [\text{CPCT}]$$

Now in $\triangle AOB$ and $\triangle COD$,

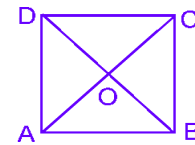
$$AB = DC \quad [\text{Sides of a square}]$$

$$\angle AOB = \angle COD \quad [\text{Vertically opposite angles}]$$

$$\angle OAB = \angle OCD \quad [\text{Alternate angles}]$$

$$\therefore \triangle AOB \cong \triangle COD \quad [\text{AAS congruence}]$$

$$\angle AO = \angle OC \quad [\text{CPCT}]$$



Similarly by taking $\triangle AOD$ and $\triangle BOC$, we can show that $OB = OD$.

$$\text{In } \triangle ABC, \angle BAC + \angle BCA = 90^\circ \quad [\because \angle B = 90^\circ]$$

$$\Rightarrow 2\angle BAC = 90^\circ \quad [\angle BAC = \angle BCA, \text{ as } BC = AD]$$

$$\Rightarrow \angle BCA = 45^\circ \quad \text{or } \angle BCO = 45^\circ$$

$$\text{Similarly } \angle CBO = 45^\circ$$

In $\triangle BCO$.

$$\angle BCO + \angle CBO + \angle BOC = 180^\circ$$

$$\Rightarrow 90^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 90^\circ$$

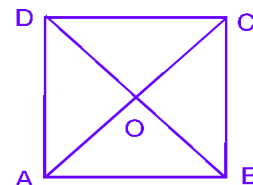
$$\Rightarrow BO \perp OC \Rightarrow BO \perp AC$$

Hence, $AC = BD$, $AC \perp BD$, $AO = OC$ and $OB = OD$. **Proved.**

Q.5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Sol. Given : A quadrilateral ABCD, in which diagonals AC and BD are equal and bisect each other at right angles,

To Prove : ABCD is a square.



Proof : Since ABCD is a quadrilateral whose diagonals bisect each other, so it is a parallelogram. Also, its diagonals bisect each other at right angles, therefore, ABCD is a rhombus.

$$\Rightarrow AB = BC = CD = DA \quad \text{[Sides of a rhombus]}$$

In $\triangle ABC$ and $\triangle BAD$, we have

$$AB = AB \quad \text{[Common]}$$

$$BC = AD \quad \text{[Sides of a rhombus]}$$

$$AC = BD \quad \text{[Given]}$$

$$\therefore \triangle ABC \cong \triangle BAD \quad \text{[SSS congruence]}$$

$$\therefore \angle ABC = \angle BAD \quad \text{[CPCT]}$$

$$\text{But, } \angle ABC + \angle BAD = 180^\circ \quad \text{[Consecutive interior angles]}$$

$$\angle ABC = \angle BAD = 90^\circ$$

$$\angle A = \angle B = \angle C = \angle D = 90^\circ \quad \text{[Opposite angles of a } \parallel^{\text{gm}} \text{]}$$

\Rightarrow ABCD is a rhombus whose angles are of 90° each.

Hence, ABCD is a square. **Proved.**

Q.6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig.). Show that

(i) it bisects $\angle C$ also,

(ii) ABCD is a rhombus.

Given : A parallelogram ABCD, in which diagonal AC bisects $\angle A$, i.e., $\angle DAC = \angle BAC$.

To Prove : (i) Diagonal AC bisects $\angle C$ i.e., $\angle DCA = \angle BCA$

(ii) ABCD is a rhombus.

Proof : (i) $\angle DAC = \angle BCA$

$$\angle BAC = \angle DCA$$

$$\text{But, } \angle DAC = \angle BAC$$

$$\therefore \angle BCA = \angle DCA$$

Hence, AC bisects $\angle DCB$

Or, AC bisects $\angle C$ **Proved.**

(ii) In $\triangle ABC$ and $\triangle CDA$

$$AC = AC \quad \text{[Common]}$$

$$\angle BAC = \angle DAC \quad \text{[Given]}$$

$$\text{and } \angle BCA = \angle DCA \quad \text{[Proved above]}$$

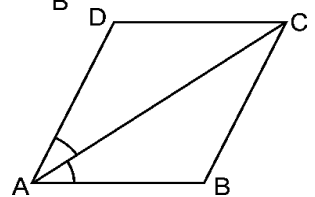
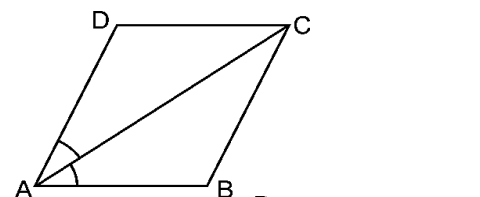
$$\therefore \triangle ABC \cong \triangle ADC \quad \text{[ASA congruence]}$$

$$\therefore BC = DC \quad \text{[CPCT]}$$

$$\text{But } AB = DC \quad \text{[Given]}$$

$$\therefore AB = BC = DC = AD$$

Hence, ABCD is a rhombus **Proved.**



[Alternate angles]

[Alternate angles]

[Given]

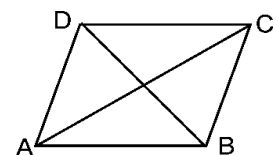
[\therefore opposite angles are equal]

Q.7. ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol. Given : ABCD is a rhombus, i.e.,

$$AB = BC = CD = DA.$$

To Prove : $\angle DAC = \angle BAC$,



$$\angle BCA = \angle DCA$$

$$\angle ADB = \angle CDB, \angle ABD = \angle CBD$$

Proof : In $\triangle ABC$ and $\triangle CDA$, we have

$$AB = AD \quad \text{[Sides of a rhombus]}$$

$$AC = AC \quad \text{[Common]}$$

$$BC = CD \quad \text{[Sides of a rhombus]}$$

$$\triangle ABC \cong \triangle ADC \quad \text{[SSS congruence]}$$

$$\text{So, } \left. \begin{array}{l} \angle DAC = \angle BAC \\ \angle BCA = \angle DCA \end{array} \right\} \text{ [CPCT]}$$

Similarly, $\angle ADB = \angle CDB$ and $\angle ABD = \angle CBD$.

Hence, diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$. **Proved.**

Q.8. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that :

(i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol. Given : ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.

To Prove : (i) ABCD is a square.
(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.

Proof : (i) In $\triangle ABC$ and $\triangle ADC$, we have
 $\angle BAC = \angle DAC$ [Given]
 $\angle BCA = \angle DCA$ [Given]
 $AC = AC$

$$\therefore \triangle ABC \cong \triangle ADC \quad \text{[ASA congruence]}$$

$$\therefore AB = AD \text{ and } CB = CD \quad \text{[CPCT]}$$

But, in a rectangle opposite sides are equal,

i.e., $AB = DC$ and $BC = AD$

$$\therefore AB = BC = CD = DA$$

Hence, ABCD is a square **Proved.**

(ii) In $\triangle ABD$ and $\triangle CDB$, we have

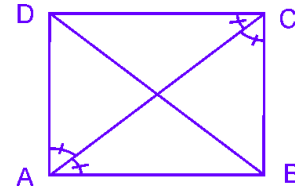
$$\left. \begin{array}{l} AD = CD \\ AB = CB \end{array} \right\} \text{ [Sides of a square]}$$

$$BD = BD \quad \text{[Common]}$$

$$\therefore \triangle ABD \cong \triangle CBD \quad \text{[SSS congruence]}$$

$$\text{So, } \left. \begin{array}{l} \angle ABD = \angle CBD \\ \angle ADB = \angle CDB \end{array} \right\} \text{ [CPCT]}$$

Hence, diagonal BD bisects $\angle B$ as well as $\angle D$ **Proved.**



Q.9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$ (see Fig.). Show that :

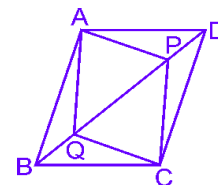
(i) $\triangle APD \cong \triangle CQB$

(ii) $AP = CQ$

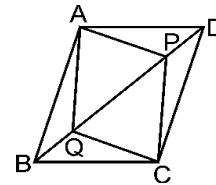
(iii) $\triangle AQB \cong \triangle CPD$

(iv) $AQ = CP$

(v) APCQ is a parallelogram



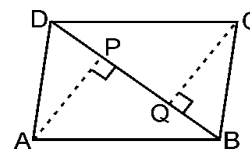
Sol. Given : ABCD is a parallelogram and P and Q are points on diagonal BD such that DP = BQ.



- To Prove :** (i) $\triangle APD \cong \triangle CQB$
(ii) $AP = CQ$
(iii) $\triangle AQB \cong \triangle CPD$
(iv) $AQ = CP$
(v) APCQ is a parallelogram

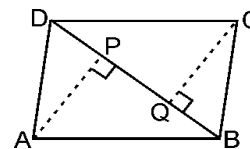
Proof : (i) In $\triangle APD$ and $\triangle CQB$, we have
 $AD = BC$ [Opposite sides of a || gm]
 $DP = BQ$ [Given]
 $\angle ADP = \angle CBQ$ [Alternate angles]
 $\therefore \triangle APD \cong \triangle CQB$ [SAS congruence]
(ii) $\therefore AP = CQ$ [CPCT]
(iii) In $\triangle AQB$ and $\triangle CPD$, we have
 $AB = CD$ [Opposite sides of a || gm]
 $DP = BQ$ [Given]
 $\angle ABQ = \angle CDP$ [Alternate angles]
 $\therefore \triangle AQB \cong \triangle CPD$ [SAS congruence]
(iv) $\therefore AQ = CP$ [CPCT]
(v) Since in APCQ, opposite sides are equal, therefore it is a parallelogram. **Proved.**

Q.10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that



- (i) $\triangle APB \cong \triangle CQD$
(ii) $AP = CQ$

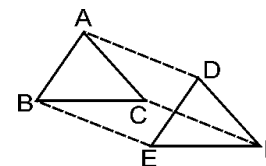
Sol. Given : ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.



- To Prove :** (i) $\triangle APB \cong \triangle CQD$
(ii) $AP = CQ$

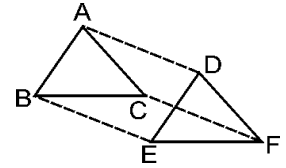
Proof : (i) In $\triangle APB$ and $\triangle CQD$, we have
 $\angle ABP = \angle CDQ$ [Alternate angles]
 $AB = CD$ [Opposite sides of a parallelogram]
 $\angle APB = \angle CQD$ [Each = 90°]
 $\therefore \triangle APB \cong \triangle CQD$ [ASA congruence]
(ii) So, $AP = CQ$ [CPCT] **Proved.**

Q.11. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig.). Show that



- (i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) $AD \parallel CF$ and $AD = CF$
(iv) quadrilateral ACFD is a parallelogram
(v) $AC = DF$
(vi) $\triangle ABC \cong \triangle DEF$

Sol. Given : In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F .



- To Prove :**
- (i) $ABED$ is a parallelogram
 - (ii) $BEFC$ is a parallelogram
 - (iii) $AD \parallel CF$ and $AD = CF$
 - (iv) $ACFD$ is a parallelogram
 - (v) $AC = DF$
 - (vi) $\triangle ABC \cong \triangle DEF$

Proof :

- (i) In quadrilateral $ABED$, we have

$$AB = DE \text{ and } AB \parallel DE. \quad [\text{Given}]$$

$\Rightarrow ABED$ is a parallelogram.

[One pair of opposite sides is parallel and equal]

- (ii) In quadrilateral $BEFC$, we have

$$BC = EF \text{ and } BC \parallel EF \quad [\text{Given}]$$

$\Rightarrow BEFC$ is a parallelogram.

[One pair of opposite sides is parallel and equal]

- (iii) $BE = CF$ and $BE \parallel CF$ [BEFC is parallelogram]

$$AD = BE \text{ and } AD \parallel BE \quad [ABED \text{ is a parallelogram}]$$

$\Rightarrow AD = CF$ and $AD \parallel CF$

- (iv) $ACFD$ is a parallelogram.

[One pair of opposite sides is parallel and equal]

- (v) $AC = DF$ [Opposite sides of parallelogram $ACFD$]

- (vi) In $\triangle ABC$ and $\triangle DEF$, we have

$$AB = DE \quad [\text{Given}]$$

$$BC = EF \quad [\text{Given}]$$

$$AC = DF \quad [\text{Proved above}]$$

$$\therefore \triangle ABC \cong \triangle DEF \quad [\text{SSS axiom}] \quad \text{Proved.}$$

Q.12. $ABCD$ is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see Fig.).

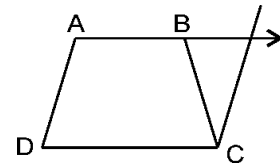
Show that

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) diagonal $AC =$ diagonal BD



Sol. Given : In trapezium $ABCD$, $AB \parallel CD$ and $AD = BC$.

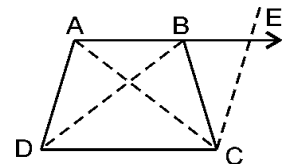
To Prove : (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) diagonal $AC =$ diagonal BD

Constructions : Join AC and BD . Extend AB and draw a line through C parallel to DA meeting AB produced at E .



Proof :

- (i) Since $AB \parallel DC$
 $\Rightarrow AE \parallel DC$... (i)
and $AD \parallel CE$... (ii) [Construction]
 $\Rightarrow ADCE$ is a parallelogram [Opposite pairs of sides are parallel]
 $\angle A + \angle E = 180^\circ$... (iii) [Consecutive interior angles]
 $\angle B + \angle CBE = 180^\circ$... (iv) [Linear pair]
 $AD = CE$... (v) [Opposite sides of a \parallel^m]
 $AD = BC$... (vi) [Given]
 $\Rightarrow BC = CE$ [From (v) and (vi)]
 $\Rightarrow \angle E = \angle CBE$... (vii) [Angles opposite to equal sides]
 $\therefore \angle B + \angle E = 180^\circ$... (viii) [From (iv) and (vii)]
Now from (iii) and (viii) we have
 $\angle A + \angle E = \angle B + \angle E$
 $\Rightarrow \angle A = \angle B$ **Proved.**
- (ii) $\left. \begin{array}{l} \angle A + \angle D = 180^\circ \\ \angle B + \angle C = 180^\circ \end{array} \right\}$ [Consecutive interior angles]
 $\Rightarrow \angle A + \angle D = \angle B + \angle C$ [$\because \angle A = \angle B$]
 $\Rightarrow \angle D = \angle C$
Or $\angle C = \angle D$ **Proved.**
- (iii) In $\triangle ABC$ and $\triangle BAD$, we have
 $AD = BC$ [Given]
 $\angle A = \angle B$ [Proved]
 $AB = CD$ [Common]
 $\therefore \triangle ABC \cong \triangle BAD$ [ASA congruence]
- (iv) diagonal $AC =$ diagonal BD [CPCT] **Proved.**

EXERCISE 8.2

Q.1. $ABCD$ is a quadrilateral in which P , Q , R and S are mid-points of the sides AB , BC , CD and DA respectively. (see Fig.). AC is a diagonal. Show that :

(i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$

(ii) $PQ = SR$

(iii) $PQRS$ is a parallelogram.

Given : $ABCD$ is a quadrilateral in which P , Q , R and S are mid-points of AB , BC , CD and DA . AC is a diagonal.

To Prove : (i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$

(ii) $PQ = SR$

(iii) $PQRS$ is a parallelogram

Proof : (i) In $\triangle ABC$, P is the mid-point of AB and Q is the mid-point of BC .

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad \dots(1)$$

[Mid-point theorem]

In $\triangle ADC$, R is the mid-point of CD and S is the mid-point of AD

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2}AC \quad \dots(2)$$

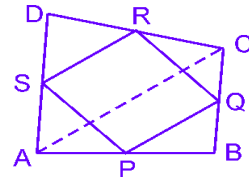
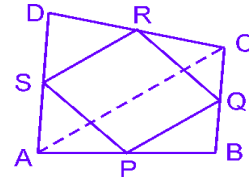
[Mid-point theorem]

(ii) From (1) and (2), we get

$$PQ \parallel SR \text{ and } PQ = SR$$

(iii) Now in quadrilateral $PQRS$, its one pair of opposite sides PQ and SR is equal and parallel.

$\therefore PQRS$ is a parallelogram. **Proved.**



Q.2. *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.*

Sol. Given : ABCD is a rhombus in which P, Q, R and S are mid points of sides AB, BC, CD and DA respectively :

To Prove : PQRS is a rectangle.

Construction : Join AC, PR and SQ.

Proof : In $\triangle ABC$

P is mid point of AB [Given]

Q is mid point of BC [Given]

$\Rightarrow PQ \parallel AC$ and $PQ = \frac{1}{2} AC$... (i) [Mid point theorem]

Similarly, in $\triangle DAC$,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$... (ii)

From (i) and (ii), we have $PQ \parallel SR$ and $PQ = SR$

\Rightarrow PQRS is a parallelogram

[One pair of opposite sides is parallel and equal]

Since ABQS is a parallelogram

$\Rightarrow AB = SQ$ [Opposite sides of a \parallel gm]

Similarly, since PBCR is a parallelogram.

$\Rightarrow BC = PR$

Thus, $SQ = PR$ [AB = BC]

Since SQ and PR are diagonals of parallelogram PQRS, which are equal.

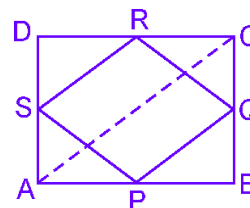
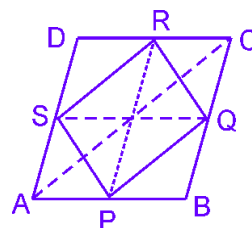
\Rightarrow PQRS is a rectangle. **Proved.**

Q.3. *ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.*

Sol. Given : A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively, PQ, QR, RS and SP are joined.

To Prove : PQRS is a rhombus.

Construction : Join AC



Proof : In $\triangle ABC$, P and Q are the mid-points of the sides AB and BC.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(i) \quad [\text{Mid point theorem}]$$

Similarly, in $\triangle ADC$,

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots(ii)$$

From (i) and (ii), we get

$$PQ \parallel SR \text{ and } PQ = SR \quad \dots(iii)$$

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal [From (iii)]

\therefore PQRS is a parallelogram.

$$\text{Now } AD = BC \quad \dots(iv)$$

[Opposite sides of a rectangle ABCD]

$$\therefore \frac{1}{2} AD = \frac{1}{2} BC$$

$$\Rightarrow AS = BQ$$

In $\triangle APS$ and $\triangle BPQ$

$$AP = BP$$

$$AS = BQ$$

$$\angle PAS = \angle PBQ$$

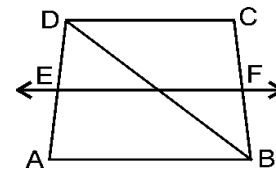
$$\triangle APS \cong \triangle BPQ$$

$$\therefore PS = PQ \quad \dots(v)$$

From (iii) and (v), we have

PQRS is a rhombus **Proved.**

Q.4. ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig.). Show that F is the mid-point of BC.



Sol. Given : A trapezium ABCD with $AB \parallel DC$, E is the mid-point of AD and $EF \parallel AB$.

To Prove : F is the mid-point of BC.

Proof : $AB \parallel DC$ and $EF \parallel AB$

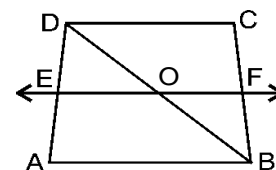
\Rightarrow AB, EF and DC are parallel.

Intercepts made by parallel lines AB, EF and DC on transversal AD are equal.

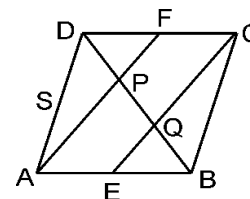
\therefore Intercepts made by those parallel lines on transversal BC are also equal.

i.e., $BF = FC$

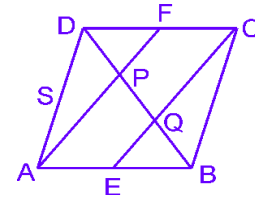
\Rightarrow F is the mid-point of BC.



Q.5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig.). Show that the line segments AF and EC trisect the diagonal BD.



Sol. Given : A parallelogram ABCD, in which E and F are mid-points of sides AB and DC respectively.



To Prove : $DP = PQ = QB$

Proof : Since E and F are mid-points of AB and DC respectively.

$$\Rightarrow AE = \frac{1}{2}AB \text{ and } CF = \frac{1}{2}DC \quad \dots(i)$$

$$\text{But, } AB = DC \text{ and } AB \parallel DC \quad \dots(ii)$$

[Opposite sides of a parallelogram]

$$\therefore AE = CF \text{ and } AE \parallel CF.$$

\Rightarrow AECF is a parallelogram.

[One pair of opposite sides is parallel and equal]

In $\triangle BAP$,

E is the mid-point of AB

$EQ \parallel AP$

\Rightarrow Q is mid-point of PB

[Converse of mid-point theorem]

$\Rightarrow PQ = QB$

...(iii)

Similarly, in $\triangle DQC$,

P is the mid-point of DQ

$DP = PQ$

...(iv)

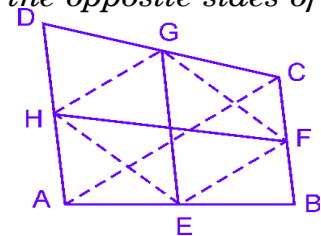
From (iii) and (iv), we have

$$DP = PQ = QB$$

or line segments AF and EC trisect the diagonal BD. **Proved.**

Q.6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol. Given : ABCD is a quadrilateral in which EG and FH are the line segments joining the mid-points of opposite sides.



To Prove : EG and FH bisect each other.

Construction : Join EF, FG, GH, HE and AC.

Proof : In $\triangle ABC$, E and F are mid-points of AB and BC respectively.

$$\therefore EF = \frac{1}{2}AC \text{ and } EF \parallel AC \quad \dots(i)$$

In $\triangle ADC$, H and G are mid-points of AD and CD respectively.

$$\therefore HG = \frac{1}{2}AC \text{ and } HG \parallel AC \quad \dots(ii)$$

From (i) and (ii), we get

$$EF = HG \text{ and } EF \parallel HG$$

\therefore EFGH is a parallelogram.

[\because a quadrilateral is a parallelogram if its one pair of opposite sides is equal and parallel]

Now, EG and FH are diagonals of the parallelogram EFGH.

\therefore EG and FH bisect each other.

[Diagonal of a parallelogram bisect each other] **Proved.**

Q.7. *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*

(i) *D is the mid-point of AC.*

(ii) *MD ⊥ AC*

(iii) *CM = MA = $\frac{1}{2}$ AB*

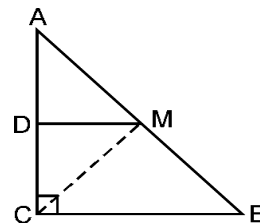
Sol. Given : A triangle ABC, in which $\angle C = 90^\circ$ and M is the mid-point of AB and $BC \parallel DM$.

To Prove : (i) D is the mid-point of AC

[Given]

(ii) $DM \perp BC$

(iii) $CM = MA = \frac{1}{2}AB$



Construction : Join CM.

Proof : (i) In $\triangle ABC$,

M is the mid-point of AB.

[Given]

$BC \parallel DM$

[Given]

D is the mid-point of AC

[Converse of mid-point theorem] **Proved.**

(ii) $\angle ADM = \angle ACB$ [\because Corresponding angles]

But $\angle ACB = 90^\circ$

[Given]

$\therefore \angle ADM = 90^\circ$

But $\angle ADM + \angle CDM = 180^\circ$

[Linear pair]

$\therefore \angle CDM = 90^\circ$

Hence, $MD \perp AC$ **Proved.**

(iii) $AD = DC$...(1) [\because D is the mid-point of AC]

Now, in $\triangle ADM$ and $\triangle CMD$, we have

$\angle ADM = \angle CDM$ [Each = 90°]

$AD = DC$ [From (1)]

$DM = DM$ [Common]

$\therefore \triangle ADM \cong \triangle CMD$ [SAS congruence]

$\Rightarrow CM = MA$...(2) [CPCT]

Since M is mid-point of AB,

$\therefore MA = \frac{1}{2}AB$...(3)

Hence, $CM = MA = \frac{1}{2}AB$ **Proved.** [From (2) and (3)]