

# 7 TRIANGLES

## EXERCISE 7.1

**Q.1.** In quadrilateral  $ACBD$ ,  
 $AC = AD$  and  $AB$  bisects  $\angle A$   
 (see Fig.). Show that  $\triangle ABC \cong \triangle ABD$ . What can  
 you say about  $BC$  and  $BD$ ?

**Sol.** In  $\triangle ABC$  and  $\triangle ABD$ , we have

$$AC = AD \quad \text{[Given]}$$

$$\angle CAB = \angle DAB$$

[Q.  $AB$  bisects  $\angle A$ ]

[Common]

$$AB = AB$$

$$\therefore \triangle ABC \cong \triangle ABD.$$

[By SAS congruence] **Proved.**

Therefore,  $BC = BD$ . (CPCT). **Ans.**

**Q.2.**  $ABCD$  is a quadrilateral in which  $AD = BC$   
 and  $\angle DAB = \angle CBA$  (see Fig.). Prove that

(i)  $\triangle ABD \cong \triangle BAC$

(ii)  $BD = AC$

(iii)  $\angle ABD = \angle BAC$

**Sol.** In the given figure,  $ABCD$  is a quadrilateral in  
 which  $AD = BC$  and  $\angle DAB = \angle CBA$ .

In  $\triangle ABD$  and  $\triangle BAC$ , we have

$$AD = BC \quad \text{[Given]}$$

$$\angle DAB = \angle CBA \quad \text{[Given]}$$

$$AB = AB \quad \text{[Common]}$$

$$\therefore \triangle ABD \cong \triangle BAC \quad \text{[By SAS congruence]}$$

$$\therefore BD = AC \quad \text{[CPCT]}$$

$$\text{and } \angle ABD = \angle BAC \quad \text{[CPCT]}$$

**Proved**

**Q.3.**  $AD$  and  $BC$  are equal perpendiculars to  
 a line segment  $AB$  (see Fig.). Show that  
 $CD$  bisects  $AB$ .

**Sol.** In  $\triangle AOD$  and  $\triangle BOC$ , we have,

$$\angle AOD = \angle BOC$$

[Vertically opposite angles]

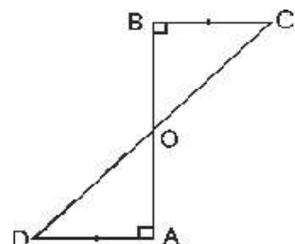
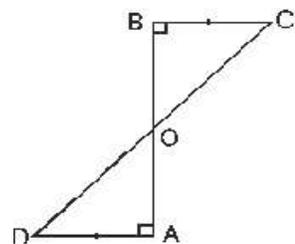
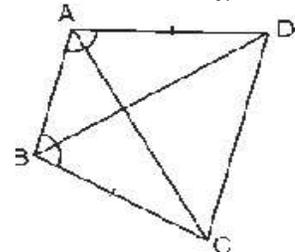
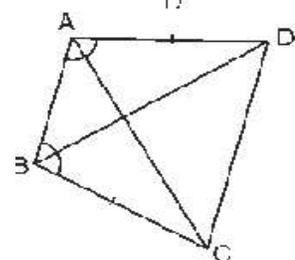
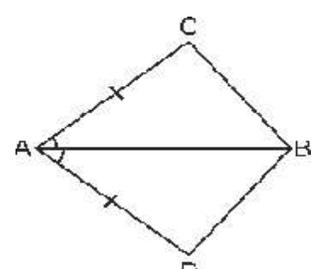
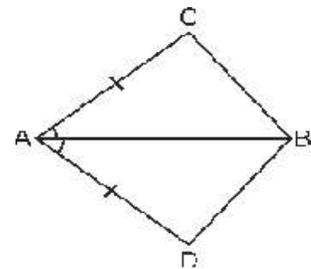
$$\angle CBO = \angle DAO \quad \text{[Each} = 90^\circ]$$

$$\text{and } AD = BC \quad \text{[Given]}$$

$$\therefore \triangle AOD \cong \triangle BOC \quad \text{[By AAS congruence]}$$

$$\text{Also, } AO = BO \quad \text{[CPCT]}$$

Hence,  $CD$  bisects  $AB$  **Proved.**



**Q.4.**  $l$  and  $m$  are two parallel lines intersected by another pair of parallel lines  $p$  and  $q$  (see Fig.). Show that  $\triangle ABC \cong \triangle CDA$ .

**Sol.** In the given figure, ABCD is a parallelogram in which AC is a diagonal, i.e.,  $AB \parallel DC$  and  $BC \parallel AD$ .

In  $\triangle ABC$  and  $\triangle CDA$ , we have,

$$\angle BAC = \angle DCA \quad [\text{Alternate angles}]$$

$$\angle BCA = \angle DAC$$

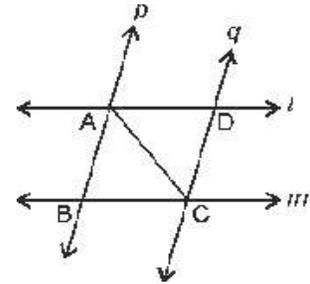
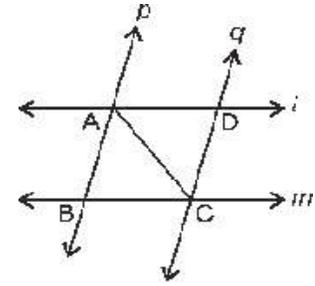
[Alternate angles]

$$AC = AC$$

[Common]

$$\therefore \triangle ABC \cong \triangle CDA \quad [\text{By ASA congruence}]$$

**Proved.**



**Q.5.** Line  $l$  is the bisector of an angle  $A$  and  $B$  is any point on  $l$ .  $BP$  and  $BQ$  are perpendiculars from  $B$  to the arms of  $\angle A$  (see Fig.). Show that :

(i)  $\triangle APB \cong \triangle AQB$

(ii)  $BP = BQ$  or  $B$  is equidistant from the arms of  $\angle A$ .

**Sol.** In  $\triangle APB$  and  $\triangle AQB$ , we have

$$\angle PAB = \angle QAB$$

[ $l$  is the bisector of  $\angle A$ ]

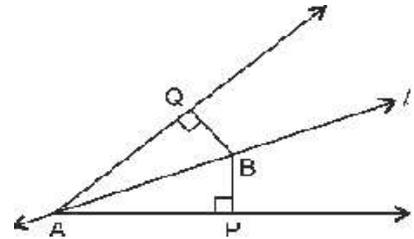
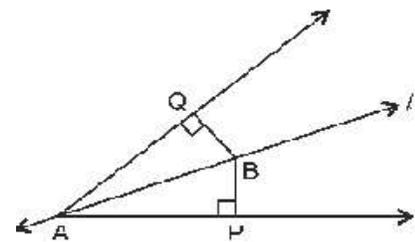
$$\angle APB = \angle AQB \quad [\text{Each} = 90^\circ]$$

$$AB = AB \quad [\text{Common}]$$

$$\therefore \triangle APB \cong \triangle AQB \quad [\text{By AAS congruence}]$$

$$\text{Also, } BP = BQ \quad [\text{By CPCT}]$$

i.e.,  $B$  is equidistant from the arms of  $\angle A$ . **Proved**



**Q.6.** In the figure,  $AC = AE$ ,  $AB = AD$  and  $\angle BAD = \angle EAC$ . Show that  $BC = DE$ .

**Sol.**  $\angle BAD = \angle EAC$  [Given]

$$\Rightarrow \angle BAD + \angle DAC = \angle EAC + \angle DAC$$

[Adding  $\angle DAC$  to both sides]

$$\Rightarrow \angle BAC = \angle EAD \quad \dots (i)$$

Now, in  $\triangle ABC$  and  $\triangle ADE$ , we have

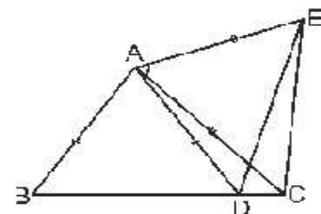
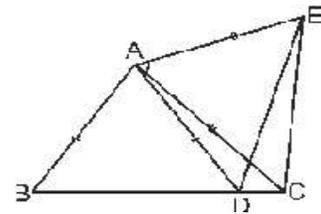
$$AB = AD \quad [\text{Given}]$$

$$AC = AE \quad [\text{Given}]$$

$$\Rightarrow \angle BAC = \angle DAE \quad [\text{From (i)}]$$

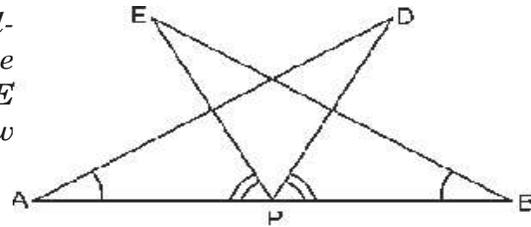
$$\therefore \triangle ABC \cong \triangle ADE \quad [\text{By SAS congruence}]$$

$$\Rightarrow BC = DE. \quad [\text{CPCT}] \quad \text{Proved.}$$



**Q.7.**  $AB$  is a line segment and  $P$  is its mid-point.  $D$  and  $E$  are points on the same side of  $AB$  such that  $\angle BAD = \angle ABE$  and  $\angle EPA = \angle DPB$  (see Fig.). Show that

- (i)  $\triangle DAP \cong \triangle EBP$  (ii)  $AD = BE$



**Sol.** In  $\triangle DAP$  and  $\triangle EBP$ , we have

$AP = BP$  [Q  $P$  is the mid-point of line segment  $AB$ ]

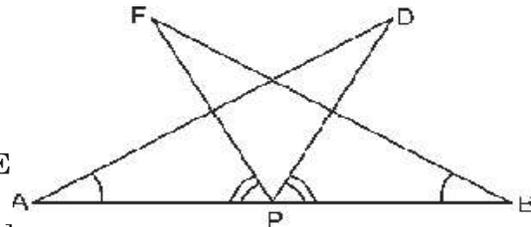
$\angle BAD = \angle ABE$  [Given]

$\angle EPB = \angle DPA$

[Q  $\angle EPA = \angle DPB \Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE$ ]

$\therefore \triangle DPA \cong \triangle EPB$  [ASA]

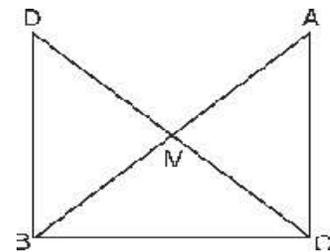
$\Rightarrow AD = BE$  [By CPCT] **Proved.**



**Q.8.** In right triangle  $ABC$ , right angled at  $C$ ,  $M$  is the mid-point of hypotenuse  $AB$ .  $C$  is joined to  $M$  and produced to a point  $D$  such that  $DM = CM$ . Point  $D$  is joined to point  $B$  (see Fig.). Show that :

- (i)  $\triangle AMC \cong \triangle BMD$   
 (ii)  $\angle DBC$  is a right angle.  
 (iii)  $\triangle DBC \cong \triangle ACB$

(iv)  $CM = \frac{1}{2}AB$



**Sol.** In  $\triangle BMB$  and  $\triangle DMC$ , we have

- (i)  $DM = CM$  [Given]

$BM = AM$

[Q  $M$  is the mid-point of  $AB$ ]

$\angle DMB = \angle AMC$

[Vertically opposite angles]

$\therefore \triangle AMC \cong \triangle BMD$  [By SAS]

**Proved.**

- (ii)  $AC \parallel BD$  [Q  $\angle DBM$  and  $\angle CAM$  are alternate angles]

$\Rightarrow \angle DBC + \angle ACB = 180^\circ$  [Sum of co-interior angles]

[Q  $\angle ACB = 90^\circ$ ] **Proved.**

$\Rightarrow \angle DBC = 90^\circ$  **Proved.**

- (iii) In  $\triangle DBC$  and  $\triangle ACB$ , we have

$DB = AC$  [CPCT]

$BC = BC$  [Common]

$\angle DBC = \angle ACB$  [Each =  $90^\circ$ ]

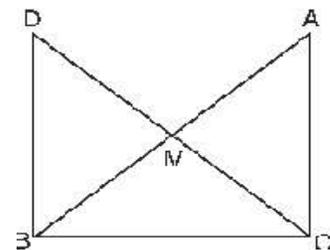
$\therefore \triangle DBC \cong \triangle ACB$  [By SAS] **Proved.**

- (iv)  $\therefore AB = CD$  [CPCT]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

Hence,  $\frac{1}{2}AB = CM$

[  $CM = \frac{1}{2}CD$ ] **Proved.**



# 7 | TRIANGLES

## EXERCISE 7.2

**Q.1.** In an isosceles triangle  $ABC$ , with  $AB = AC$ , the bisectors of  $\angle B$  and  $\angle C$  intersect each other at  $O$ . Join  $A$  to  $O$ . Show that :

(i)  $OB = OC$  (ii)  $AO$  bisects  $\angle A$ .

**Sol.** (i)  $AB = AC \Rightarrow \angle ABC = \angle ACB$

[Angles opposite to equal sides are equal]

$$\frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB$$

$$\Rightarrow \angle CBO = \angle BCO$$

[  $OB$  and  $OC$  are bisectors of  $\angle B$  and  $\angle C$  respectively]

$$\Rightarrow OB = OC \quad [\text{Sides opposite to equal angles are equal}]$$

$$\text{Again, } \angle \frac{1}{2} ABC = \angle \frac{1}{2} ACB$$

$$\Rightarrow \angle ABO = \angle ACO \quad [ \because OB \text{ and } OC \text{ are bisectors of } \angle B \text{ and } \angle C \text{ respectively}]$$

In  $\triangle ABO$  and  $\triangle ACO$ , we have

$$AB = AC$$

[Given]

$$OB = OC$$

[Proved above]

$$\angle ABO = \angle ACO$$

[Proved above]

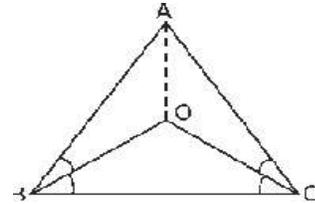
$$\therefore \triangle ABO \cong \triangle ACO$$

[SAS congruence]

$$\Rightarrow \angle BAO = \angle CAO$$

[CPCT]

$\Rightarrow AO$  bisects  $\angle A$  **Proved.**



**Q.2.** In  $\triangle ABC$ ,  $AD$  is the perpendicular bisector of  $BC$  (see Fig.). Show that  $\triangle ABC$  is an isosceles triangle in which  $AB = AC$ .

**Sol.** In  $\triangle ABD$  and  $\triangle ACD$ , we have

$$\angle ADB = \angle ADC \quad [\text{Each} = 90^\circ]$$

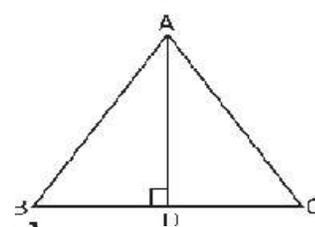
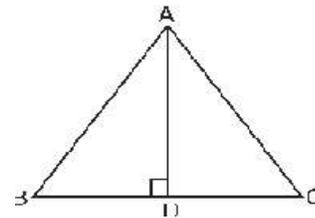
$$BD = CD \quad [Q \text{ AD bisects } BC]$$

$$AD = AD \quad [\text{Common}]$$

$$\therefore \triangle ABD \cong \triangle ACD \quad [\text{SAS}]$$

$$\therefore AB = AC \quad [\text{CPCT}]$$

Hence,  $\triangle ABC$  is an isosceles triangle. **Proved.**



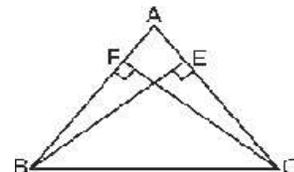
**Q.3.**  $ABC$  is an isosceles triangle in which altitudes  $BE$  and  $CF$  are drawn to equal sides  $AC$  and  $AB$  respectively (see Fig.). Show that these altitudes are equal.

**Sol.** In  $\triangle ABC$ ,

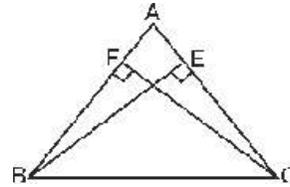
$$AB = AC \quad [\text{Given}]$$

$$\Rightarrow \angle B = \angle C \quad [\text{Angles opposite to equal sides of a triangle are equal}]$$

Now, in right triangles  $BFC$  and  $CEB$ ,



$\angle BFC = \angle CEB$  [Each =  $90^\circ$ ]  
 $\angle FBC = \angle ECB$  [Proved above]  
 $BC = BC$  [Common]  
 $\therefore \triangle BFC \cong \triangle CEB$  [AAS]  
 Hence,  $BE = CF$  [CPCT] **Proved.**



**Q.4.**  $ABC$  is a triangle in which altitudes  $BE$  and  $CF$  to sides  $AC$  and  $AB$  are equal (see Fig.). Show that

(i)  $\triangle ABE \cong \triangle ACF$

(ii)  $AB = AC$ , i.e.,  $ABC$  is an isosceles triangle.

**Sol.** (i) In  $\triangle ABE$  and  $\triangle ACF$ , we have

$BE = CF$  [Given]

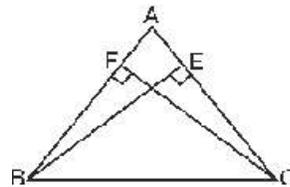
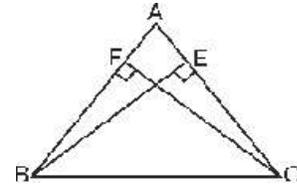
$\angle BAE = \angle CAF$  [Common]

$\angle BEA = \angle CFA$  [Each =  $90^\circ$ ]

So,  $\triangle ABE \cong \triangle ACF$  [AAS] **Proved.**

(ii) Also,  $AB = AC$  [CPCT]

i.e.,  $ABC$  is an isosceles triangle **Proved.**



**Q.5.**  $ABC$  and  $DBC$  are two isosceles triangles on the same base  $BC$  (see Fig.). Show that  $\angle ABD = \angle ACD$ .

**Sol.** In isosceles  $\triangle ABC$ , we have

$AB = AC$

$\angle ABC = \angle ACB$  ... (i)

[Angles opposite to equal sides are equal]

Now, in isosceles  $\triangle DCB$ , we have

$BD = CD$

$\angle DBC = \angle DCB$  ... (ii)

[Angles opposite to equal sides are equal]

Adding (i) and (ii), we have

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow \angle ABD = \angle ACD$  **Proved.**

**Q.6.**  $\triangle ABC$  is an isosceles triangle in which  $AB = AC$ . Side  $BA$  is produced to  $D$  such that  $AD = AB$  (see Fig.). Show that  $\angle BCD$  is a right angle.

**Sol.**  $AB = AC$  [Given]

$\angle ACB = \angle ABC$  ... (i)

[Angles opposite to equal sides are equal]

$AB = AD$  [Given]

$\therefore AD = AC$  [Q  $AB = AC$ ]

$\therefore \angle ACD = \angle ADC$  ... (ii) [Angles opposite to equal sides are equal]

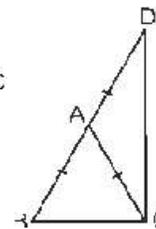
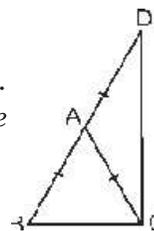
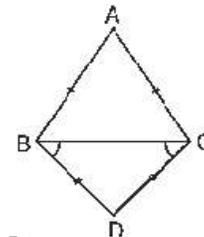
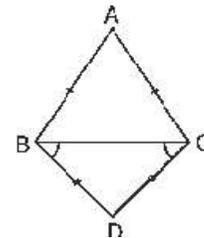
Adding (i) and (ii)

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow \angle BCD = \angle ABC + \angle ADC$  ... (iii)

Now, in  $\triangle BCD$ , we have

$\angle BCD + \angle DBC + \angle BDC = 180^\circ$  [Angle sum property of a triangle]



$$\therefore \angle BCD + \angle BCD = 180^\circ$$

$$\Rightarrow 2\angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = 90^\circ$$

Hence,  $\angle BCD = 90^\circ$  or a right angle **Proved.**

**Q.7.** *ABC is a right angled triangle in which  $\angle A = 90^\circ$  and  $AB = AC$ . Find  $\angle B$  and  $\angle C$ .*

**Sol.** In  $\triangle ABC$ , we have

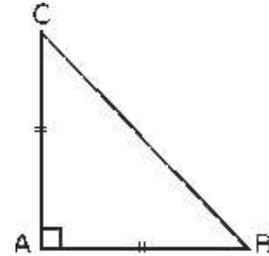
$$\left. \begin{array}{l} \angle A = 90^\circ \\ \text{and } AB = AC \end{array} \right\} \text{ [Given]}$$

We know that angles opposite to equal sides of an isosceles triangle are equal.

So,  $\angle B = \angle C$

Since,  $\angle A = 90^\circ$ , therefore sum of remaining two angles =  $90^\circ$ .

Hence,  $\angle B = \angle C = 45^\circ$  **Answer.**



**Q.8.** *Show that the angles of an equilateral triangle are  $60^\circ$  each.*

**Sol.** As  $\triangle ABC$  is an equilateral.

$$\text{So, } AB = BC = AC$$

$$\text{Now, } AB = AC$$

$$\Rightarrow \angle ACB = \angle ABC \quad \dots(i)$$

[Angles opposite to equal sides of a triangle are equal]

$$\text{Again, } BC = AC$$

$$\Rightarrow \angle BAC = \angle ABC \quad \dots(ii) \quad \text{[same reason]}$$

Now, in  $\triangle ABC$ ,

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ \quad \text{[Angle sum property of a triangle]}$$

$$\Rightarrow \angle ABC + \angle ABC + \angle ABC = 180^\circ \quad \text{[From (i) and (ii)]}$$

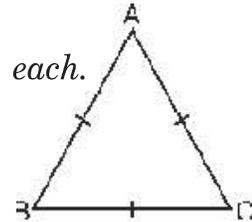
$$\Rightarrow 3 \angle ABC = 180^\circ$$

$$\Rightarrow \angle ABC = \frac{180^\circ}{3} = 60^\circ$$

Also, from (i) and (ii)

$$\angle ACB = 60^\circ \text{ and } \angle BAC = 60^\circ$$

Hence, each angle of an equilateral triangle is  $60^\circ$  **Proved.**



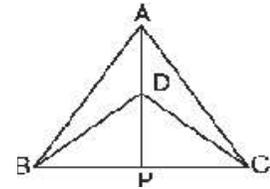
## 7

## TRIANGLES

## EXERCISE 7.3

**Q.1.**  $\triangle ABC$  and  $\triangle DBC$  are two isosceles triangles on the same base  $BC$  and vertices  $A$  and  $D$  are on the same side of  $BC$  (see Fig.). If  $AD$  is extended to intersect  $BC$  at  $P$ , show that

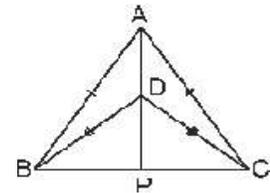
- (i)  $\triangle ABD \cong \triangle ACD$
- (ii)  $\triangle ABP \cong \triangle ACP$
- (iii)  $AP$  bisects  $\angle A$  as well as  $\angle D$ .
- (iv)  $AP$  is the perpendicular bisector of  $BC$ .



**Sol.** (i) In  $\triangle ABD$  and  $\triangle ACD$ , we have

$$\begin{aligned} AB &= AC && \text{[Given]} \\ BD &= CD && \text{[Given]} \\ AD &= AD && \text{[Common]} \\ \therefore \triangle ABD &\cong \triangle ACD && \text{[SSS congruence]} \end{aligned}$$

**Proved.**



(ii) In  $\triangle ABP$  and  $\triangle ACP$ , we have

$$\begin{aligned} AB &= AC && \text{[Given]} \\ \angle BAP &= \angle CAP && \text{[Q } \angle BAD = \angle CAD, \text{ by CPCT]} \\ AP &= AP && \text{[Common]} \\ \therefore \triangle ABP &\cong \triangle ACP && \text{[SAS congruence]} \end{aligned}$$

**Proved.**

(iii)  $\triangle ABD \cong \triangle ADC$  [From part (i)]

$$\Rightarrow \angle ADB = \angle ADC \quad \text{[CPCT]}$$

$$\Rightarrow 180^\circ - \angle ADB = 180^\circ - \angle ADC$$

$$\Rightarrow \text{Also, from part (ii), } \angle BAPD = \angle CAP \quad \text{[CPCT]}$$

$\therefore AP$  bisects  $\angle A$  as well as  $\angle D$ . **Proved.**

(iv) Now,  $BP = CP$

$$\text{and } \angle BPA = \angle CPA \quad \text{[By CPCT]}$$

$$\text{But } \angle BPA + \angle CPA = 180^\circ \quad \text{[Linear pair]}$$

$$\text{So, } 2\angle BPA = 180^\circ$$

$$\text{Or, } \angle BPA = 90^\circ$$

Since  $BP = CP$ , therefore  $AP$  is perpendicular bisector of  $BC$ .

**Proved.**

**Q.2.**  $AD$  is an altitude of an isosceles triangle  $ABC$  in which  $AB = AC$ . Show that

- (i)  $AD$  bisects  $BC$       (ii)  $AD$  bisects  $\angle A$ .

**Sol.** (i) In  $\triangle ABD$  and  $\triangle ACD$ , we have

$$\angle ADB = \angle ADC \quad [\text{Each} = 90^\circ]$$

$$AB = AC \quad [\text{Given}]$$

$$AD = AD \quad [\text{Common}]$$

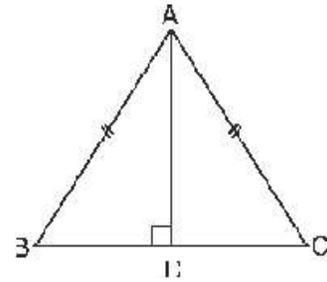
$$\therefore \triangle ABD \cong \triangle ACD \quad [\text{RHS congruence}]$$

$$\therefore BD = CD \quad [\text{CPCT}]$$

Hence,  $AD$  bisects  $BC$ .

- (ii) Also,  $\angle BAD = \angle CAD$

Hence  $AD$  bisects  $\angle A$  **Proved**



**Q.3.** Two sides  $AB$  and  $BC$  and median  $AM$  of one triangle  $ABC$  are respectively equal to sides  $PQ$  and  $QR$  and median  $PN$  of  $\triangle PQR$  (see Fig.). Show that :

- (i)  $\triangle ABM \cong \triangle PQN$       (ii)  $\triangle ABC \cong \triangle PQR$

**Sol.** (i) In  $\triangle ABM$  and  $\triangle PQN$ ,  
we have

$$BM = QN$$

$$[BC = QR]$$

$$\Rightarrow \frac{1}{2} BC = \frac{1}{2} QR$$

$$AB = PQ \quad [\text{Given}]$$

$$AM = PN \quad [\text{Given}]$$

$$\therefore \triangle ABM \cong \triangle PQN \quad [\text{SSS congruence}]$$

$$\Rightarrow \angle ABM = \angle PQN \quad [\text{CPCT}]$$

**Proved.**

- (ii) Now, in  $\triangle ABC$  and  $\triangle PQR$ , we have

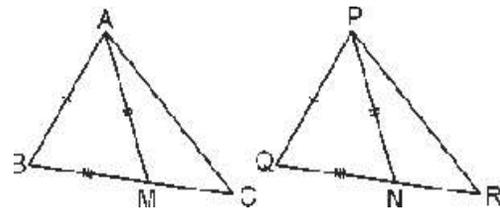
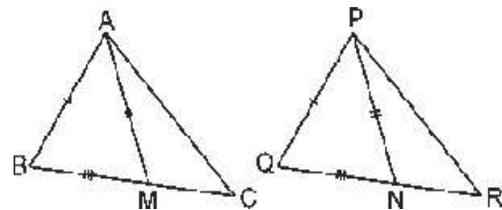
$$AB = PQ \quad [\text{Given}]$$

$$\angle ABC = \angle PQR \quad [\text{Proved above}]$$

$$BC = QR \quad [\text{Given}]$$

$$\therefore \triangle ABC \cong \triangle PQR \quad [\text{SAS congruence}]$$

**Proved.**



**Q.4.** *BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.*

**Sol.** BE and CF are altitudes of a  $\triangle ABC$ .

$$\therefore \angle BEC = \angle CFB = 90^\circ$$

Now, in right triangles BEC and CFB, we have

$$\text{Hyp. BC} = \text{Hyp. BC} \quad [\text{Common}]$$

$$\text{Side BE} = \text{Side CF} \quad [\text{Given}]$$

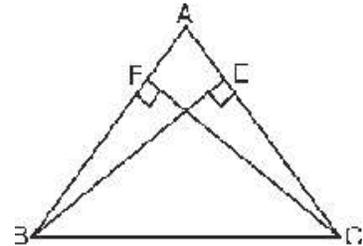
$$\therefore \triangle BEC \cong \triangle CFB \quad [\text{By RHS congruence rule}]$$

$$\therefore \angle BCE = \angle CBF \quad [\text{CPCT}]$$

Now, in  $\triangle ABC$ ,  $\angle B = \angle C$

$$\therefore AB = AC \quad [\text{Sides opposite to equal angles are equal}]$$

Hence,  $\triangle ABC$  is an isosceles triangle. **Proved.**



**Q.5.** *ABC is an isosceles triangle with  $AB = AC$ . Draw  $AP \perp BC$  to show that  $\angle B = \angle C$ .*

**Sol.** Draw  $AP \perp BC$ .

In  $\triangle ABP$  and  $\triangle ACP$ , we have

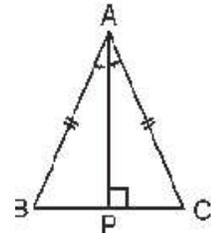
$$AB = AC \quad [\text{Given}]$$

$$\angle APB = \angle APC \quad [\text{Each} = 90^\circ]$$

$$AP = AP \quad [\text{Common}]$$

$$\therefore \triangle ABP \cong \triangle ACP \quad [\text{By RHS congruence rule}]$$

Also,  $\angle B = \angle C$  **Proved.** [CPCT]



# 7 TRIANGLES

## EXERCISE 7.4

**Q.1.** Show that in a right angled triangle, the hypotenuse is the longest side.

**Sol.** ABC is a right triangle, right angled at B.

Now,  $\angle A + \angle C = 90^\circ$

$\Rightarrow$  Angles A and C are each less than  $90^\circ$ .

Now,  $\angle B > \angle A$

$\Rightarrow AC > BC$  ... (i)

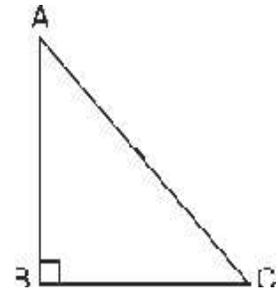
[Side opposite to greater angle is longer]

Again,  $\angle B > \angle C$

$\Rightarrow AC > AB$  ... (ii)

[Side opposite to greater angle is longer]

Hence, from (i) and (ii), we can say that AC (Hypotenuse) is the longest side. **Proved**



**Q.2.** In the figure, sides AB and AC of  $\triangle ABC$  are extended to points P and Q respectively. Also,  $\angle PBC < \angle QCB$ . Show that  $AC > AB$ .

**Sol.**  $\angle ABC + \angle PBC = 180^\circ$  [Linear pair]

$\Rightarrow \angle ABC = 180^\circ - \angle PBC$  ... (i)

Similarly,  $\angle ACB = 180^\circ - \angle QCB$  ... (ii)

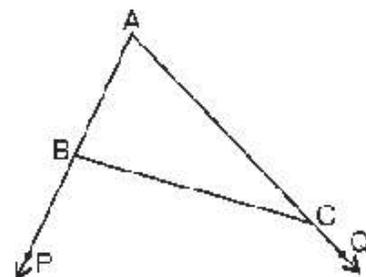
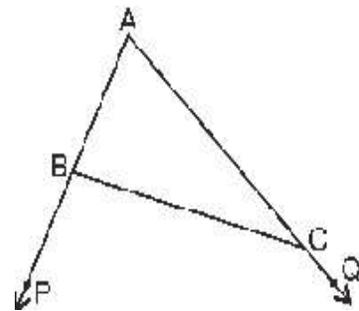
It is given that  $\angle PBC < \angle QCB$

$\therefore 180^\circ - \angle QCB < 180^\circ - \angle PBC$

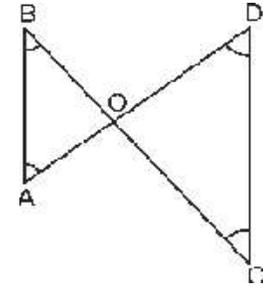
Or  $\angle ACB < \angle ABC$  [From (i) and (ii)]

$\Rightarrow AB < AC$

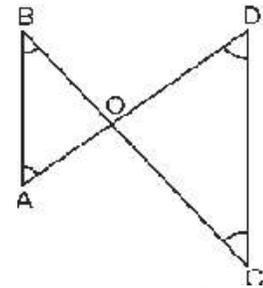
$\Rightarrow AC > AB$  **Proved.**



**Q.3.** In the figure,  $\angle B < \angle A$  and  $\angle C < \angle D$ . Show that  $AD < BC$ .



**Sol.**  $\angle B < \angle A$  [Given]  
 $BO > AO$  ... (i)  
 [Side opposite to greater angle is longer]  
 $\angle C < \angle D$  [Given]  
 $\Rightarrow CO > DO$  ... (ii)  
 [Same reason]  
 Adding (i) and (ii)  
 $BO + CO > AO + DO$   
 $\Rightarrow BC > AD$   
 $AD < BC$  **Proved.**



**Q.4.**  $AB$  and  $CD$  are respectively the smallest and longest sides of a quadrilateral  $ABCD$  (see Fig.). Show that  $\angle A > \angle C$  and  $\angle B > \angle D$ .

**Sol.** Join  $AC$ .

Mark the angles as shown in the figure.

In  $\triangle ABC$ ,

$BC > AB$  [AB is the shortest side]

$\Rightarrow \angle 2 > \angle 4$  ... (i)

[Angle opposite to longer side is greater]

In  $\triangle ADC$ ,

$CD > AD$  [CD is the longest side]

$\angle 1 > \angle 3$  ... (ii)

[Angle opposite to longer side is greater]

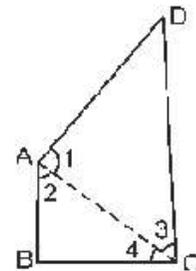
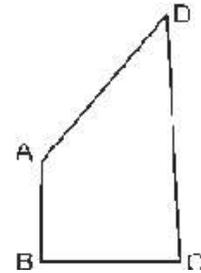
Adding (i) and (ii), we have

$\angle 2 + \angle 1 > \angle 4 + \angle 3$

$\Rightarrow \angle A > \angle C$  **Proved.**

Similarly, by joining  $BD$ , we can prove that

$\angle B > \angle D$ .



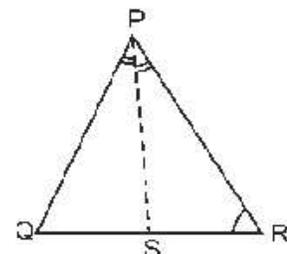
**Q.5.** In the figure,  $PR > PQ$  and  $PS$  bisects  $\angle QPR$ . Prove that  $\angle PSR > \angle PSQ$ .

**Sol.**  $PR > PQ$

$\angle PQR > \angle PRQ$  ... (i)

[Angle opposite to longer side is greater]

$\angle QPS > \angle RPS$  [ $\because PS$  bisects  $\angle QPR$ ] ... (ii)



In  $\triangle PQS$ ,  $\angle PQS + \angle QPS + \angle PSQ = 180^\circ$

$$\Rightarrow \angle PSQ = 180^\circ - (\angle PQS + \angle QPS) \quad \dots(\text{iii})$$

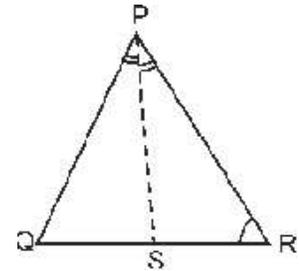
Similarly in  $\triangle PRS$ ,  $\angle PRS + \angle RPS + \angle PSR = 180^\circ$

$$\Rightarrow \angle PSR = 180^\circ - (\angle PRS + \angle RPS) \quad [\text{from (ii) ... (iv)}]$$

From (i), we know that  $\angle PQS < \angle PRS$

So from (iii) and (iv),  $\angle PSQ < \angle PSR$

$$\Rightarrow \angle PSR > \angle PSQ \quad \text{Proved}$$



**Q.6.** Show that of all the segments drawn from a given point not on it, the perpendicular line segment is the shortest.

**Sol.** We have a line  $\overleftrightarrow{l}$  and O is a point not on  $\overleftrightarrow{l}$ .

$$OP \perp \overleftrightarrow{l}$$

We have to prove that  $OP < OQ$ ,  $OP < OR$  and  $OP < OS$ .

In  $\triangle OPQ$ ,  $\angle P = 90^\circ$

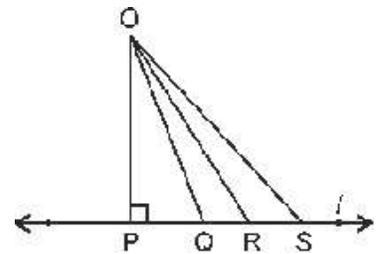
$\therefore \angle Q$  is an acute angle (i.e.,  $\angle Q < 90^\circ$ )

$\therefore \angle Q < \angle P$

Hence,  $OP < OQ$

[Side opposite to greater angle is longer]

Similarly, we can prove that  $OP$  is shorter than  $OR$ ,  $OS$  etc. **Proved.**

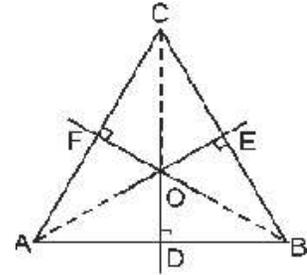


# 7 TRIANGLES

## EXERCISE 7.5 (OPTIONAL)

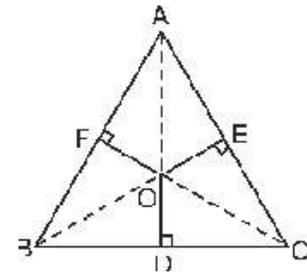
**Q.1.**  $ABC$  is a triangle. Locate a point in the interior of  $\triangle ABC$  which is equidistant from all the vertices of  $\triangle ABC$ .

**Sol.** Draw perpendicular bisectors of sides  $AB$ ,  $BC$  and  $CA$ , which meets at  $O$ . Hence,  $O$  is the required point.



**Q.2.** In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

**Sol.**



**Q.3.** In a huge park, people are concentrated at three points (see Fig.).

$A$  : where there are different slides and swings for children,

$B$  : near which a man-made lake is situated,

$C$  : which is near to a large parking and exit.

Where should an icecream parlour be set up so that maximum number of persons can approach it?

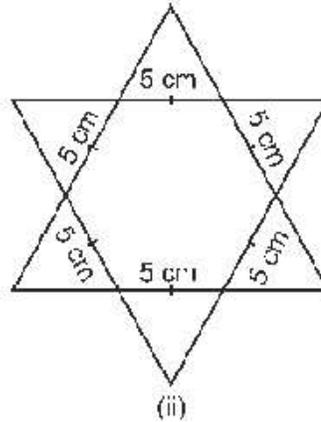
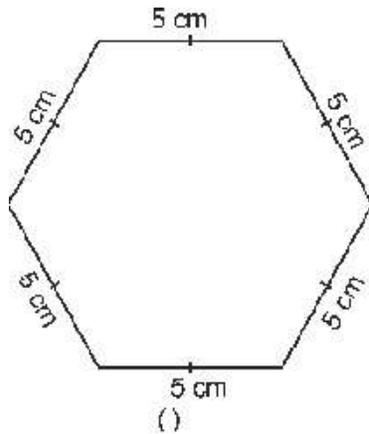


Draw bisectors  $\angle A$ ,  $\angle B$  and  $\angle C$  of  $\triangle ABC$ . Let these angle bisectors meet at  $O$ .

$O$  is the required point.

**Sol.** Join  $AB$ ,  $BC$  and  $CA$  to get a triangle  $ABC$ . Draw the perpendicular bisector of  $AB$  and  $BC$ . Let them meet at  $O$ . Then  $O$  is equidistant from  $A$ ,  $B$  and  $C$ . Hence, the icecream pra

**Q.4.** Complete the hexagonal and star shaped Rangolies [see Fig. (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



**Sol.**

